Comment on "Hitting probabilities of diffusion-limited-aggregation clusters"

A. P. Roberts and M. A. Knackstedt
Department of Applied Mathematics, Research School of Physical Sciences and Engineering, Australian National University, Canberra, Australian Capital Territory 0200, Australia
(Received 26 September 1991; revised manuscript received 26 April 1993)

In a recent paper Wolf [Phys. Rev. A **43**, 5504 (1991)] proposes an alternative method for calculating the growth-site probability distribution (GSPD) of diffusion-limited-aggregation (DLA) clusters. When comparing the technique with other available methods, Wolf assumes that the models of DLA and dielectric breakdown are analogous. By calculating the GSPD for DLA using a correct electrostatic analogy we demonstrate the effectiveness of Wolf's method.

PACS number(s): 68.70.+w, 05.40.+j

The study of simple irreversible-growth models has led to a greater understanding of the physical origin of fractal growth processes. In recent years attention has focused on the growth-site probability distribution (GSPD) as an effective way to characterize these growth processes [1–4].

In a recent paper Wolf [5(a)] presented an alternative method utilizing a theorem of Spitzer [5(b)] to calculate the GSPD for diffusion-limited-aggregation [6] (DLA) clusters. Wolf gauges the success of this method by comparing the GSPD of a 65-particle cluster calculated using Spitzer's theorem, with those obtained both by Monte Carlo simulation and by solving the Laplace equation. He only finds a rough qualitative agreement between the methods. In this Comment we show that the probability distribution Wolf obtains by solving the Laplace equation incorporates the dielectric-breakdown-model [7] (DBM) boundary conditions rather than DLA boundary conditions, and that a growth rule is employed which does not correspond to either the DLA or DBM model. When the appropriate boundary conditions and growth rule are used Wolf's method compares very favorably over all ranges of the GSPD indicating that the technique is successful. Finally we question a result of Wolf which gives a different probability distribution when standard and nonstandard boundary conditions are used.

In DLA growth [6] a Brownian particle is launched from a random position on a circle of "large" diameter which encloses the cluster. In the on-lattice version of DLA, growth occurs when the particle reaches a site adjacent to the cluster (a *surface* site). A new particle is then launched and the process repeated. The probability that a particle occupies a lattice site (i,j) $p_{i,j}$ satisfies the discrete Laplace equation [8] $\nabla^2 p_{i,j}=0$. The boundary conditions on the field are $p_{i,j}=\text{const}$ on a far circular boundary to simulate an isotropic flux and $p_{i,j}=0$ at the surface sites of the cluster. The probability that growth occurs at the kth surface site P_k is thus

$$P_k \propto \sum_{i,j} p_{i,j} \ , \tag{1}$$

where the sum is over sites (i,j) in the probability field adjacent to the kth surface site. In the electrostatic analogy of DLA the *surface* sites adjacent to the cluster and the enclosing boundary (electrode) are held at different constant potentials ϕ_1 and ϕ_2, and the field satisfies the discrete Laplace equation $\nabla^2\phi_{i,j}^{\text{DLA}}=0$ in the interior. The growth probability is given by

$$P_k \propto \sum_{i,j} |\nabla\phi_{i,j}^{\text{DLA}}| \ . \tag{2}$$

In contrast to DLA, DBM [7] has the *cluster* sites and the enclosing boundary held at different constant potentials. Moreover, in DBM, the growth probability is given by

$$P_k \propto N|\nabla\phi_{i,j}^{\text{DBM}}|^{\eta} \ , \tag{3}$$

where N is the number of occupied *cluster* sites adjacent to the *surface* site k with $\phi_{i,j}^{\text{DBM}}$ again satisfying the discrete Laplace equation.

Wolf uses the DBM boundary conditions to evaluate the GSPD of the model cluster (shown in Fig. 1) and a growth rule of the form $P_k \propto |\nabla\phi_{i,j}^{\text{DBM}}|$. He then quantitatively compares this GSPD with the hitting probabilities obtained using the Spitzer theorem and a Monte Carlo

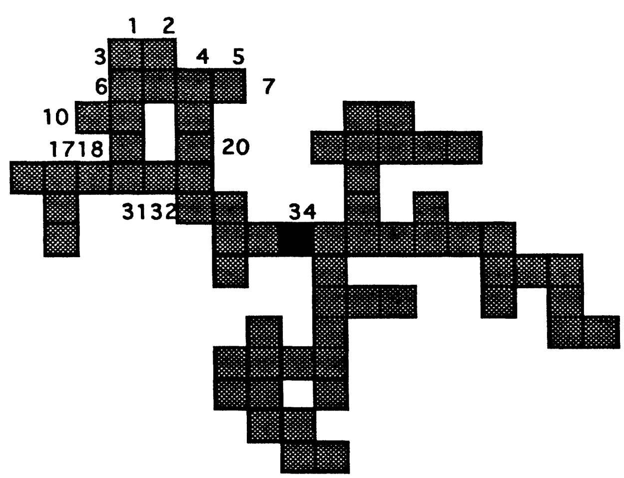

FIG. 1. The 65-particle cluster generated by Wolf (Fig. 17 in Ref. [5(a)]).

 ©1993 The American Physical Society

TABLE I. Growth probabilities for the perimeter sites of the cluster shown in Fig. 1. The second column contains the values generated by Wolf for DLA using the theorem of Spitzer. The third column contains the normalized probability distribution calculated with the correct boundary condition and growth rule on a circle with diameter 121 lattice units. Column 4 is from Wolf's work and column 5 is the probability distribution for DBM boundary conditions and growth rule. $x[y]$ denotes $x \times 10^y$.

Site number	Theorem of Spitzer	Electrostatic analogy of DLA (This work)	DLA (Wolf)	DBM (This work)
1	4.302[−2]	4.298[−2]	3.710[−2]	3.100[−2]
2	3.498[−2]	3.494[−2]	3.338[−2]	2.788[−2]
3	3.554[−2]	3.550[−2]	2.946[−2]	2.461[−2]
4	1.071[−2]	1.070[−2]	1.465[−2]	2.448[−2]
5	2.411[−2]	2.409[−2]	2.137[−2]	1.786[−2]
6	1.060[−2]	1.058[−2]	1.632[−2]	2.727[−2]
7	1.397[−2]	1.396[−2]	1.428[−2]	1.193[−2]
10	2.330[−2]	2.328[−2]	2.253[−2]	1.882[−2]
17	0.000[0]	0.000[0]	1.305[−2]	1.090[−2]
18	0.000[0]	0.000[0]	3.262[−3]	8.172[−3]
20	9.662[−5]	9.656[−5]	1.719[−3]	1.437[−3]
31	1.433[−3]	1.432[−3]	3.803[−3]	3.181[−3]
32	1.086[−3]	1.085[−3]	2.718[−3]	4.546[−3]

method. We compare selected values of the GSPD obtained by Wolf's method based on Spitzer's theorem with the results found by solving the Laplace equation using the correct electrostatic analogy for DLA [given by Eq. (1)]. The results are presented in Table I. The GSPD is calculated with the potential at the cluster equal to 1 and the potential at the outer boundary set at zero. As can be seen in Table I, Wolf's technique for calculating the GSPD is in excellent agreement with our solution of the Laplace equation using the correct electrostatic analogy of DLA.

The GSPD for DBM, given in column 5 of Table I, can be calculated using the growth rule given by (3). Note the ratio of the probabilities from our DBM calculation to those of Wolf's DLA calculation is a constant (0.836) for the case where a site has just 1 cluster neighbor (e.g., site 1 of Fig. 1) and 2 and 3 times that factor for the case of 2 (site 6) and 3 (site 18) cluster neighbors, respectively. This indicates that Wolf employs the growth rule $P_k \propto |\nabla \phi_{i,j}^{\mathrm{DBM}}|$.

The result of Wolf which gives a different GSPD when the values of the potential at the boundaries are varied is also questioned. In DBM and the electrostatic analogy of DLA, the cluster, and the enclosing boundary can be set at different arbitrary potentials. A simple argument shows this will not affect the relative magnitude of the growth probabilities and the solutions can be shown to be unique [9]. This is contrary to calculations presented in columns 3 and 4 of Table II in Wolf's paper for a DLA cluster. It is unlikely that numerical imprecision could account for the differences which are of the order of 10%. Thus this calculation appears to be in error.

The success of Wolf's technique for calculating the growth-site probability distribution for DLA has been demonstrated. In concluding, we emphasize the need for care when interchanging the lattice models of DLA and DBM. The seemingly small difference in the local growth rule and local boundary condition affects the morphological properties of the cluster [10] as well as the GSPD from which the multifractal properties are derived.

[1] R. Blumenfeld and A. Aharony, Phys. Rev. Lett. **62**, 2977 (1989).
[2] S. Schwarzer *et al.*, Phys. Rev. Lett. **65**, 603 (1990).
[3] J. Lee and H. E. Stanley, Phys. Rev. Lett. **61**, 2945 (1988).
[4] S. Schwarzer, J. Lee, S. Havlin, H. E. Stanley, and P. Meakin, Phys. Rev. A **43**, 1134 (1991).
[5] (a) M. Wolf, Phys. Rev. A **43**, 5504 (1991). (b) F. Spitzer, *Principles of the Random Walk* (Van Nostrand, Princeton, NJ, 1964), p. 121, Theorem 12.1 and p. 141, Theorem 14.1.
[6] T. A. Witten and L. M. Sander, Phys. Rev. B **27**, 5686 (1983).
[7] L. Niemeyer, L. Pietronero, and H. J. Wiesmann, Phys. Rev. Lett. **52**, 1033 (1984).
[8] L. Pietronero and H. J. Wiesmann, J. Stat. Phys. **36**, 909 (1984).
[9] From the *discrete* Laplace equation it is clear that the value of the potential at a site is the average of the value at its four neighbors. This means that the solution cannot have maxima or minima except on the boundaries. Hence the difference between any two solutions each satisfying identical Dirichlet boundary conditions is everywhere zero, which implies uniqueness.
[10] A. P. Roberts and M. A. Knackstedt (unpublished).